

Temperature effects on mixed-state geometric phase

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Geometric phase of an open quantum system that is interacting with a thermal environment (bath) is studied through some simple examples. The system is considered to be a simple spin-half particle which is weakly coupled to the bath. It is seen that even in this regime the geometric phase can vary with temperature. In addition, we also consider the system under an adiabatically time-varying magnetic field which is weakly coupled to the bath. An important feature of this model is that it reveals existence of a temperature-scale in which adiabaticity condition is preserved and beyond which the geometric phase is varying quite rapidly with temperature. This temperature is exactly the one in which the geometric phase vanishes. This analysis has some implications in realistic implementations of geometric quantum computation.

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I. INTRODUCTION

Geometric phase (GP) of a quantum state is one of important concepts in quantum theory. Indeed, Berry was the first who emphasized the geometric nature of the phase acquired by an eigenstate of an adiabatically-varying Hamiltonian in a closed loop in parameter space [1]. Since its discovery, this concept has been a subject of a vast investigation and generalization in many aspects [2, 3]. An important reason for the interest in the concept of GP is its relevance to geometric quantum computation [4]. Indeed, it is believed that the purely geometric characteristic of such phases potentially provides robustness against certain sources of noise [5].

For general evolutions of an open quantum system, where the dynamics is generally nonunitary, the GP has been defined in different methods [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. The first general approach is Uhlmann's mathematical method which is based on a purification of mixed states [6]. Another, more recent, approach is based on a kinematic extension of the interferometric approach firstly used in the case of unitary evolutions [13]. These methods are generally argued to be different [16], however, recently a formal approach bridging between them has also been proposed [17].

As mentioned above studying how GP is affected by environmental noises is an important issue in investigation of robustness of geometric quantum computation. In some earlier works, effect of different types of decoherence sources on GP has been studied. In Ref. [12], it has been shown that the GP of a spin-half system driven by one or two mode quantum fields subject to decoherence behaves differently for adiabatic and nonadiabatic evolutions. Also in Ref. [18], the GP of a two-level system driven by a quantized magnetic field subject to dephasing has been considered and it has been shown that the GP acquired by the system can be observed even in long time-scales. Ref. [14] addresses the change of the mean GP when the two-level system is weakly coupled to a thermal. An interesting feature of this investigation is that the mean GP does not have any thermal correction up to the first order in the coupling constant, that is, any dependence on temperature is of higher orders in the coupling constant. In Ref. [19], it has been shown that coupling to an environment induces some ge-

ometric and nongeometric corrections to the Berry phase (BP) of a spin-half system that is under an adiabatically slow rotating magnetic field. Moreover, it has been argued that the BP can be observed only in experiments whose time-scales, on one hand, are slow enough to ignore nonadiabatic correction, and fast enough, on the other hand, to prevent dephasing from deleting all phase information. Existence of such a finite (adiabaticity) time-scale for the GP, indeed, has been shown to be a general feature of open quantum systems [15, 20]. Some other related studies about decoherence effects on the GP can be found in Refs. [21, 22, 23, 24].

II. THE MODELS

In this paper we consider a simple spin-half particle in a magnetic field $\vec{B}(t)$ which is weakly coupled to an environment (a thermal bath, for example, consisting of photons) with temperature T . We are mainly interested in finding thermal effects of the environment on the GP of the system. To be specific, here, we follow the kinematic definition of the GP [13]. If the density matrix of the system evolves as $\varrho(0) \rightarrow \varrho(t) = \sum_i p_i(t) |w_i(t)\rangle \langle w_i(t)|$, where $t \in [0, \tau]$ and $p_i(t)$'s ($|w_i(t)\rangle$'s) are eigenvalues (eigenvectors) of $\varrho(t)$, then the GP acquired by this state during this interval reads as follows:

$$\Phi = \arg \sum_i [p_i(0)p_i(\tau)]^{\frac{1}{2}} \langle w_i(0) | w_i(\tau) \rangle e^{-\int_0^\tau \langle w_i(t) | \dot{w}_i(t) \rangle dt}. \quad (1)$$

Throughout this paper we always consider the GP measured in $\text{mod } 2\pi$. To use the above relation one must solve the eigenvalue problem for the density matrix of the system to find eigenvalues as well as the corresponding eigenvectors.

Before discussion about coupling of an environment and the system in a relatively realistic model, let us illustrate a couple of simple examples that can reveal some general features of more complicated models. As the first example, suppose that due to interaction with the environment, our system which is a spin-half particle, initially is prepared in the thermally impure state $(1 - \epsilon)|+\rangle\langle+| + \epsilon\varrho_{\text{th}}$, where $|+\rangle_n = \begin{pmatrix} \cos \frac{\vartheta(t)}{2} e^{-\frac{i}{2}\varphi(t)} \\ \sin \frac{\vartheta(t)}{2} e^{\frac{i}{2}\varphi(t)} \end{pmatrix}$ is an eigenvector of $\mathbf{n} \cdot \vec{\sigma}$, $\mathbf{n} =$

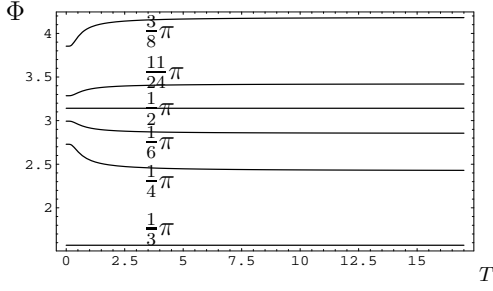


FIG. 1: GP (Φ , in radian) vs. temperature (T , in the units where $\hbar \equiv k_B \equiv 1$) for different values of ϑ . Here, we have taken $\epsilon = 1/3$ and $\Delta = 1$ (in the same natural units).

$(\sin \vartheta \cos \varphi, \sin \vartheta \cos \varphi, \cos \vartheta)$, $\varrho_{\text{th.}} = \lambda|+; 0\rangle_n \langle +; 0| + (1 - \lambda)|-; 0\rangle_n \langle -; 0|$, $\frac{\lambda}{1-\lambda} = e^{-\Delta/T}$ (we work in the natural units where $\hbar \equiv k_B \equiv 1$), Δ is the energy gap of the system, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector of the Pauli matrices, and for simplicity we take $\vartheta(t) = \vartheta(0)$ and $\varphi(t) = \omega_0 t$. Now let us assume that the density matrix of the system has the following (unitary) evolution during the period $t \in [0, \frac{2\pi}{\omega_0}]$

$$[(1 - \epsilon) + \lambda\epsilon]|+; t\rangle_n \langle +; t| + (1 - \lambda)\epsilon|-; t\rangle_n \langle -; t|. \quad (2)$$

By using the general formula (1) one can easily obtain the GP acquired by this state after its cyclic evolution as follows:

$$\Phi = \pi + \arctan(\tan(\pi \cos \vartheta)[1 - 2\epsilon/(1 + e^{-\Delta/T})]). \quad (3)$$

Figure 1 shows that depending on the value of ϑ the GP may decrease or increase with temperature. We will see a bit later that the GP shows such a typical behavior even in more complicated cases.

As the second example, let us consider two coupled spin-half particles, one of which driven by a time-varying magnetic field, with the Hamiltonian

$$H(t) = \frac{1}{2}\vec{B}(t) \cdot \vec{\sigma}_1 + J(\sigma_{+1}\sigma_{-2} + \text{H.c.}), \quad (4)$$

where $\sigma_{\pm} = \frac{1}{2}(\sigma_x \pm i\sigma_y)$ and the indices 1 and 2 indicate the corresponding subsystems. The magnetic field is considered to be $B\mathbf{n}(t)$, where $\mathbf{n}(t)$ has a form as in the previous example but it rotates about the z -axis adiabatically. In fact, in this simple case the second particle can be taken as an environment which is coupled to our system of interest. The eigenvalues (E_j) and instantaneous eigenvectors ($|\Psi_j(t)\rangle$) of this Hamiltonian can be found simply as follows:

$$E_{1(3)} = -E_{2(4)} = \sqrt{1 + \frac{\kappa^2}{2} \pm \frac{\kappa}{2}\sqrt{\kappa^2 + 4\sin^2 \vartheta}}, \quad (5)$$

$$|\Psi_j\rangle = \frac{1}{\sqrt{N_j}}(a_j|\uparrow\downarrow\rangle + b_j|\uparrow\uparrow\rangle + c_j|\downarrow\downarrow\rangle + d_j|\downarrow\uparrow\rangle), \quad (6)$$

where $d_j = \frac{\kappa \sin \vartheta (\cos \vartheta - E_j)}{E_j^2 - \cos 2\vartheta} e^{i\omega_0 t}$, $b_j = -\frac{E_j + \cos \vartheta}{\sin \vartheta} d_j e^{-i\omega_0 t}$, $a_j = \sin \vartheta e^{-i\omega_0 t}$, $c_j = E_j - \cos \vartheta$, $N_j = |a_j|^2 + b_j^2 + c_j^2 + |d_j|^2$, and $\kappa = \frac{2J}{B}$. In Ref. [25], the BP's of the subsystems for all eigenvectors have been calculated, and it has been shown that the BP of the composite system is sum of the BP's of the two constituting subsystems. Here, we suppose that

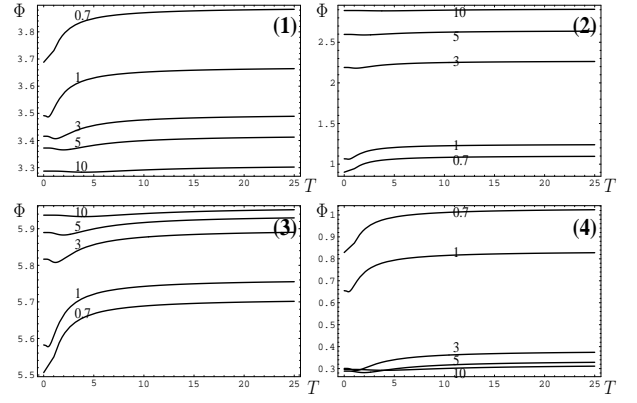


FIG. 2: BP (Φ , in radian) of the subsystem 1 vs. temperature (T , in the units where $\hbar \equiv k_B \equiv 1$), obtained from ρ_1^j , $j = 1, 2, 3, 4$, for $\vartheta = \pi/4$, with $\epsilon = 0.1$. The numbers show the related values of the coupling constant κ .

the composite system is in the thermally diluted state $\rho_j = (1 - \epsilon)|\Psi_j\rangle\langle\Psi_j| + \epsilon\rho_{\text{thermal}}$, in which $\rho_{\text{thermal}} \propto e^{-H/T}$. We are interested in calculating the BP of the subsystem 1 to see how it is affected by this simple thermalization mechanism. By a simple calculation, it can be seen that in the Schmidt decomposition of the eigenvectors coefficients are time-independent, $|\Psi_j(t)\rangle = \sum_{\alpha} \sqrt{p_{\alpha}^j} |F_{\alpha}^j(t)\rangle_1 |f_{\alpha}^j(t)\rangle_2$. This fact in turn results into the reduced density matrix $\rho_1^j(t) = \text{tr}_2(\rho_j) = \sum_{i\alpha} \tilde{p}_{\alpha}^j |F_{\alpha}^j(t)\rangle\langle F_{\alpha}^j(t)|$ for the subsystem 1, in which for $i = j$ we have $\tilde{p}_{\alpha}^j = [1 - \epsilon + \epsilon e^{-E_j/T} / \text{tr}(e^{-H/T})] p_{\alpha}^j$ and $\tilde{p}_{\alpha}^i = \epsilon e^{-E_i/T} p_{\alpha}^i / \text{tr}(e^{-H/T})$ otherwise. To calculate the BP of this density matrix we use the idea of purification

$$\Phi_{\text{Berry}}(\rho_1^j(t)) = \Phi_{\text{Berry}}(|\Upsilon^j(t)\rangle_{1a}), \quad (7)$$

where $|\Upsilon^j(t)\rangle_{1a} = \sum_{i\alpha} \sqrt{\tilde{p}_{\alpha}^j} |F_{\alpha}^j(t)\rangle_1 |a_{\alpha}^j\rangle_a$ is a purified state obtained from $\rho_1^j(t)$ by attaching an ancilla a . By using this purification and considering constancy of the coefficients of the Schmidt decompositions, it can easily be concluded that

$$\Phi_{\text{Berry}}(\rho_1^j) = \sum_{i\alpha} \tilde{p}_{\alpha}^j \Phi_{\text{Berry}}(F_{\alpha}^j), \quad (8)$$

where $\Phi_{\text{Berry}}(F_{\alpha}^j) = i \int_0^{2\pi} \langle F_{\alpha}^j(t) | \dot{F}_{\alpha}^j(t) \rangle dt$. Figure 2 illustrates the BP of the subsystem 1 during a cyclic evolution, for

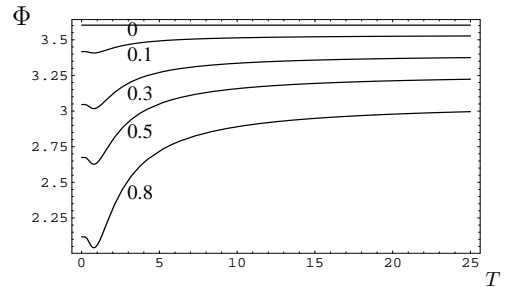


FIG. 3: BP (Φ , in radian), vs. temperature (T , in the units where $\hbar \equiv k_B \equiv 1$) for different values of ϵ . Here, we have taken ρ_1^1 , and $\kappa = 2$, $\vartheta = \pi/4$.

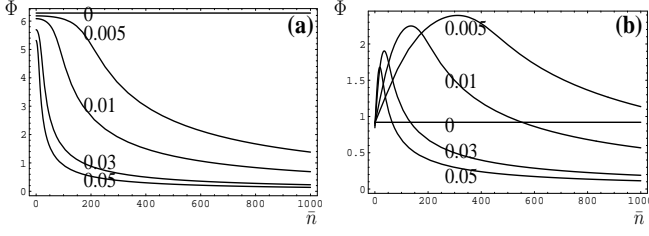


FIG. 4: GP (Φ , in radian), vs. mean number of photons (\bar{n}) in the environment. Here, $\omega_0 = 2$, $\vartheta = \frac{\pi}{2}$ (a), $\frac{\pi}{2} + \frac{\pi}{4}$ (b), and the numbers associated to each curve denote the related value of the coupling constant κ .

$j = 1, 2, 3, 4$. As is seen, after a decrease for some temperature ranges, when temperature increases the BP increases as well. Figure 3 shows that the more thermal the composite system is the lower the value of the BP is. However, even in this case the general behavior of the BP vs. temperature is as in Fig. 2.

Now, let us consider a more realistic case in which the spin is interacting with a thermal environment consisting of photons. It is known that when the spin is not isolated its dynamics is much more involved. When the magnetic field is not time-varying, interaction with the environment generally induces energy and phase relaxation processes (with the time-scales T_1 and T_2 , respectively), in addition to a Lamb shift of the energy levels (δE_{Lamb}) [26]. It is natural to expect that when dephasing is the leading regime of the interaction no GP effect could be observed [19]. However, in the weak-coupling limit, defined by $B \gg T_2^{-1}$, in experiment time-scale τ these effects can be seen if both adiabaticity condition $B \gg \tau^{-1}$ and no-dephasing regime condition $\tau \lesssim T_2$ can be satisfied simultaneously [19]. To fulfill these conditions, in our discussion we assume that the field varies adiabatically on a closed loop in the parameter space in time τ . Additionally, we also suppose that the time-scale in which typical correlation functions of the environment decay, $t_{\text{env.}}$, is much smaller than τ and all dissipation-induced time-scales, i.e., $t_{\text{env.}} \ll \tau, T_{\text{diss.}} = \min\{T_1, T_2, \delta E_{\text{Lamb}}^{-1}\}$. These assumptions overall enable us to use Markovian approximation in the form of a Lindblad-type master equation [27]

$$\dot{\rho} = -i[H, \rho] + \frac{1}{2} \sum_k \left([\Gamma_k, \rho \Gamma_k^\dagger] + [\Gamma_k \rho, \Gamma_k^\dagger] \right), \quad (9)$$

where H is the effective Hamiltonian of the (open) system and Γ_k 's are the quantum jump operators describing system-environment coupling. The magnetic field is supposed to be in the direction $\mathbf{n}(t) = (\sin \vartheta \cos \varphi(t), \sin \vartheta \sin \varphi(t), \cos \vartheta)$, where $\varphi(t) = \omega_0 t$, and $\tau = \frac{2\pi}{\omega_0}$ is the time necessary for completion of a loop. The Hamiltonian of the system is $H(t) = \frac{1}{2} \vec{B}(t) \cdot \vec{\sigma}$, where $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g| \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, in which $|g\rangle$ ($|e\rangle$) is the ground (excited) state of the system in the absence of external magnetic field. For such an open system the quantum jump operators are $\Gamma_1 = \sqrt{\kappa(\bar{n}+1)}|g\rangle\langle e|$, which describes spontaneous and stimulated emissions into the bath, and $\Gamma_2 = \sqrt{\kappa\bar{n}}|e\rangle\langle g|$, which is the corresponding absorption from the bath, where $\bar{n}(\omega_0, T) = (e^{\omega_0/T} - 1)^{-1}$ is

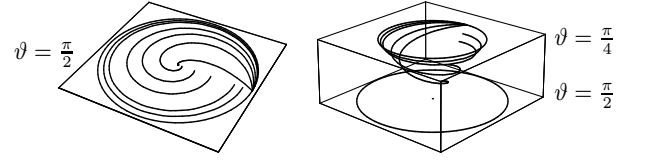


FIG. 5: The path traversed by the tip of the Bloch vector of the spin starting from $\vartheta = \frac{\pi}{2}$, for $\bar{n} = 0, 1, 5, 10, 20, 80$ (left), and $\vartheta = \pi/4$, for $\bar{n} = 0, 1, 5, 10, 20$ (right). In both of the figures, we have set $\kappa = 10^{-2}$, $B = 10^3$, and $\omega_0 = 2$.

the mean number of photons in the bath. A master equation in the Lindblad form usually can be applied in the case of time-independent Hamiltonians under Markovian conditions. The simpler case in which the Hamiltonian of the system is $\frac{1}{2}\omega_0\sigma_z$, a constant magnetic field along z -axis, has been studied earlier in Ref. [14], and to the first order in coupling constant κ , the mean GP has been calculated. It has been shown that up to this order no thermal correction appears in the GP, that is, any dependence on temperature (through \bar{n}) is of higher orders of κ . Figure 4 depicts temperature-dependence of the GP for the two values of $\vartheta = \pi/2, 3\pi/4$ and different values of the coupling constant κ . For the values $\vartheta < \frac{\pi}{2}$ the case is similar to $\vartheta = \frac{\pi}{2}$. When $\vartheta \leq \frac{\pi}{2}$, this figure shows a decrease in the GP with temperature, whereas for $\vartheta > \frac{\pi}{2}$ the GP can also show an increase for some temperature range whose size decreases when κ increases.

Now, let us consider the general case with a time-varying magnetic field, as explained above. To be able to use the Lindblad master equation we change our reference frame and go to the diagonal frame in which the Hamiltonian has the simple time-independent form $h\sigma_z$, where $h = \frac{1}{2}B\sqrt{\sin^2 \vartheta + (\cos \vartheta - \omega_0/B)^2}$ [21]. Here it is worth noting that another alternative method to consider the problem is to define an adiabatic Hamiltonian [28], which in the absence of interaction with environment shows the BP effect. In this way, in contrast to the rotating frame method, only the length of magnetic field changes. The results of this method have been shown to be equivalent to the rotating frame method in the first order of the rotation frequency [23]. The density matrix of the system in the diagonal frame, denoted by $\tilde{\rho}$, is related to the density matrix in the lab frame as

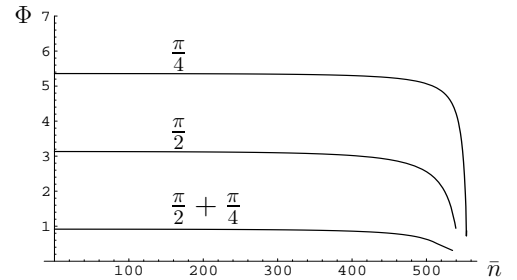


FIG. 6: GP (Φ , in radian) vs. mean number of photons (\bar{n}) in the environment. Here, $\omega_0 = 2$, $\kappa = 10^{-3}$, $B = 10^3$, and the azimuthal angles $\vartheta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{4}$ are chosen for this plot.

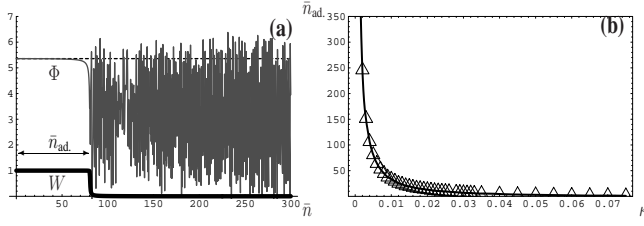


FIG. 7: (a) GP (Φ , in radian) vs. mean number of photons (\bar{n}) in environment (thin curve) and $W = |\langle w_1(0)|w_1(\tau)\rangle|$ (thick curve). Here, $\omega_0 = 2$, $B = 10^3$, $\kappa = 5 \times 10^{-3}$, $\vartheta = \frac{\pi}{4}$. (b) \bar{n}_{ad} vs. coupling constant κ . Here, we have set $\omega_0 = 2$ and $B = 10^3$.

$\varrho(t) = e^{-\frac{i}{2}t\omega_0\sigma_z} V \tilde{\varrho}(t) V e^{\frac{i}{2}t\omega_0\sigma_z}$, where $V = \alpha_- \sigma_x + \alpha_+ \sigma_z$, and $\alpha_{\pm} = \sqrt{\frac{1}{2} \pm \frac{1}{4h}(B \cos \vartheta - \omega_0)}$. The explicit form of the master equation in this frame is as follows:

$$\begin{aligned} \dot{\tilde{\varrho}} = & -i[h\sigma_z, \tilde{\varrho}] + \kappa(\bar{n} + 1)(2\sigma_- \tilde{\varrho} \sigma_+ - \tilde{\varrho} \sigma_+ \sigma_- - \sigma_+ \sigma_- \tilde{\varrho}) \\ & + \kappa\bar{n}(2\sigma_+ \tilde{\varrho} \sigma_- - \tilde{\varrho} \sigma_- \sigma_+ - \sigma_- \sigma_+ \tilde{\varrho}). \end{aligned} \quad (10)$$

For simplicity we have omitted the Lamb shift terms, because the Lamb shift contributes to the GP for $\tau \neq 2\pi/\omega_0$ only [14]. These equations can be solved exactly as follows:

$$\begin{aligned} \tilde{\varrho}_{11}(t) &= \tilde{\varrho}_{11}(0)e^{-2\kappa t(2\bar{n}+1)} + \frac{\bar{n}}{1+\bar{n}}[1 - e^{-2\kappa t(2\bar{n}+1)}] \\ \tilde{\varrho}_{12}(t) &= \tilde{\varrho}_{12}(0)e^{(-2ih + \kappa(2\bar{n}+1))t}, \end{aligned} \quad (11)$$

where $\tilde{\varrho}_{11}$ ($\tilde{\varrho}_{12}$) is the diagonal (off-diagonal) element of $\tilde{\varrho}$. We assume that the initial state of the spin is $|\Psi_0\rangle = \cos \frac{\vartheta}{2}|e\rangle + \sin \frac{\vartheta}{2}|g\rangle$. Unlike the case of unitary evolution of mixed states where the GP can be explained only by length of the Bloch vector (r) and the solid angle subtended by it on the Bloch sphere (Ω), i.e. $\Phi = -\arctan(r \tan \frac{\Omega}{2})$, in the case of nonunitary evolution such information is not sufficient and the GP has a complicated dependence on the Bloch vector. However, dynamics of the Bloch vector can give an insight about how dissipative mechanisms affect the system. Figure 5 illustrates these effects on the Bloch vector of the density matrix. As is seen, in this case the length of the Bloch vector decreases, which characterizes a coherence loss, and, the vector precesses toward the center of the sphere, which is related to dissipation. After coming back to the lab frame, to find the GP of the state one has to solve the eigenvalue problem of $\varrho(t)$, and plug the result into Eq. (1). The explicit form of the

GP is very tedious, instead, we plot the results of some special cases in Fig. (6). It shows that for a relatively large temperature range the GP is varying very slowly with temperature. This is in accordance with the results of Ref. [14]. In addition, Fig. 7(a) shows that when temperature gradually increases the GP decreases so that in some threshold value it vanishes. After this value the GP shows a rapidly varying behavior. The value of this threshold depends on the parameters of the problem. An special and important feature of this figure is that the overlap of $|\Psi_0\rangle \equiv |w_1(0)\rangle$ and $|w_1(\tau)\rangle$, $W = |\langle w_1(0)|w_1(\tau)\rangle|$, as a measure of adiabaticity of the evolution vanishes in the same threshold temperature. This indicates that after this temperature adiabaticity condition breaks down. This implies existence of a temperature-scale \bar{n}_{ad} for the observation of the adiabatic GP. This is similar to the existence of a finite adiabaticity time-scale for observation of the GP, as pointed out in Ref. [15]. In fact, this similarity between temperature and time can be understood easily by looking back at Eq. (11). Figure 7(b) shows this threshold temperature vs. the coupling constant. As expected, when the coupling constant increases the adiabaticity temperature-scale decreases.

III. CONCLUSION

We have studied thermal effects induced by an environment on the geometric phase. It has been shown that even in weak-coupling and adiabatic limits geometric phase can vary with the temperature of the environment. One of implications of the results of this paper is that in geometric phase experiments in systems which are in contact to a dissipative environment care must be taken in correct interpretation of results. The existence of an adiabaticity temperature-scale, similar to the adiabaticity time-scale, puts some constraints on the realization schemes of geometric quantum information processing. This in turn can have some implications in the more realistic investigations of robustness of geometric quantum computation [29].

Acknowledgments

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